Problems That Look Hard But Aren't

What Happens

In the final session of this unit, students work in small groups to generate a list of "hard" multiplication pairs (or relationships). The class compiles a list of these pairs and discusses which are the most difficult relationships to learn and which they have developed strategies for solving. They solve a two-digit multiplication problem as a whole class. As an assessment activity, students solve another two-digit problem in two different ways. Their work focuses on:

- applying multiplication relationships to a two-digit problem
- using what they know about multiplication to discuss and solve harder problems

Materials

- Interlocking cubes (class set available)
- One-centimeter graph paper (1–2 per student)
- Student Sheet 15 (1 per student)
- Student Sheet 16 (1 per student, homework)
- Overhead projector, transparency pen

Activity

Discussing Multiplication Pairs

Organize students into groups of four. Each group is responsible for making a list of the five multiplication pairs that are the most difficult to learn (up to 12's table). You might suggest to students that they look back to the list they made when they played Multiplication Pairs. The group needs to agree on five pairs to present to the class. Groups will need 5 to 10 minutes to prepare their lists.

Activity

Which Pairs Are Hard for You?

Each group presents the list of multiplication pairs that its members consider the most difficult to learn. There may be pairs that are common to many groups' lists. As you record these pairs on the board, put a check mark next to any that are repeated (instead of recording them more than once). From this list determine a class list of the five to eight multiplication relationships that are the most difficult to learn. These will be the ones that have the most check marks next to them. Challenge students to learn to solve these problems in the next few days.

There may be some difference of opinion about pairs some students consider hard and others do not. Have students share strategies for how they learned or remember these relationships. Some strategies students have used in the past for learning hard relationships are breaking the problem into two easier problems, relating the problem to one they already know, or picking one or two hard problems each day to learn. The **Dialogue Box**, Strategies for Learning "Hard Problems," p. 59, gives some examples of student strategies.

Activity

Problems That Look Hard But Aren't

Present a two-digit by one-digit multiplication problem such as this on the board or overhead:

31×6

Some people think this looks like a hard multiplication problem. During the past two weeks you've been developing strategies for solving these kinds of problems. I'd like you to talk briefly with the person next to you about how you would solve this problem.

Students talk with the people near them about how they would solve the problem. Bring the group together to share strategies for approaching this multiplication problem. Record students' ideas on the board. Ask students why some people might think this was a hard multiplication problem.

What are some other two-digit multiplication problems that look hard but really aren't?

Record this list on the board and save it. Students will choose one problem from the list to do for homework.

Activity

Assessment

Solving Two-Digit Multiplication Problems

Students work alone to solve the next problem. They solve the problem in two different ways and write about how they solved the problem. Graph paper and interlocking cubes should be available for those students who need them.

Pass out a copy of Student Sheet 15, Two Ways to Solve a Problem (p. 111), to each student. Tell students you would like to get an idea of how they are thinking about multiplication problems they have been doing and what kinds of strategies they are using.

These responses are one way of assessing a student's understanding of multiplication problems. See the **Teacher Note**, Two Ways to Solve 27×4 (p. 58). As you look at student papers, there are several questions you can focus on:

- Are students keeping track of their work carefully?
- Are students becoming fluent in the basic multiplication relationships?
- Do students demonstrate an understanding of partitioning large numbers into more familiar parts as a way of multiplying?
- Are students using tools (centimeter paper, cubes) appropriately to help them solve the problem?

❖ **Tip for the Linguistically Diverse Classroom** Have limited-English-proficient students place the materials they used to solve the problem on top of their completed worksheet. Have them express their thought processes by illustrating how they used the materials.

Activity

Choosing Student Work to Save

As the unit ends, you may want to use one of the following options for creating a record of students' work on this unit:

- Students look back through their folders or notebooks and write about what they learned in this unit, what they remember most, and what was hard or easy for them. Students could do this during their writing time.
- Students select one or two pieces of their work as their best, and you also choose one or two pieces of their work to be saved. This work may be saved in a portfolio for the year. You might include students' written solutions to the assessment, Creating and Solving Division Problems (Investigation 2, Sessions 7 and 8), and any other assessment tasks from this unit. Students can create a separate page with brief comments describing the pieces of work.
- You may want to send a selection of work home for parents to see. Students write a cover letter describing their work in this unit. This work should be returned if you are keeping a year-long portfolio of mathematics work for each student.

Session 5 Follow-Up

Homework

Most likely, all students will not have had the opportunity to finish every choice item. You might decide to have these choices remain in the classroom for students to work on during their free time or for homework.

Note: Students will need to use their sets of arrays later in the year in the unit *Packages and Groups*. We suggest students keep their bags of arrays in their current math folders and continue to use them as a way of learning the basic multiplication relationships. You will also need to save array transparencies.

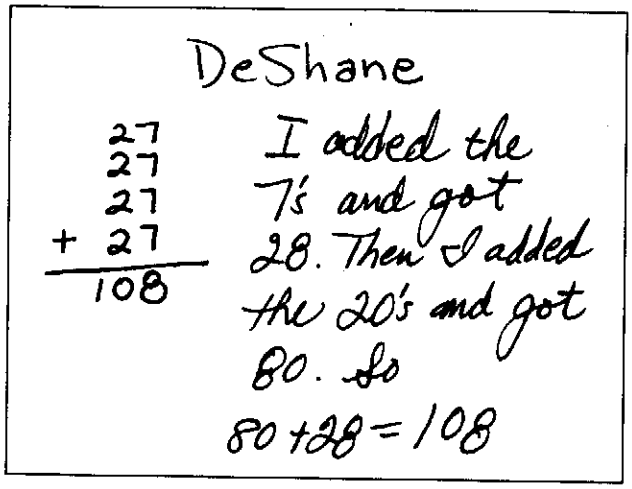
Problems That Look Hard But Aren't On Student Sheet 16, students choose one multiplication problem from the class list of "Problems That Look Hard But Aren't" to solve and write about.

Teacher Note

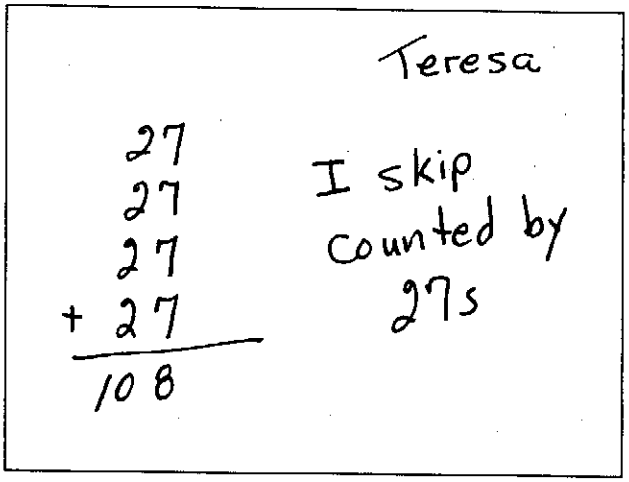
Two Ways to Solve 27 × 4

As you look at students' work for this assessment, consider whether each student has developed reliable strategies and can record clearly the steps used to solve the problem. Has the student used a combination of addition and multiplication strategies? Can the student pull apart a "hard" multiplication problem into smaller related multiplication problems? Asking students to solve problems in more than one way helps them to think flexibly and also gives them a way to check their work. The following are examples of students' solutions to the problem 27 × 4.

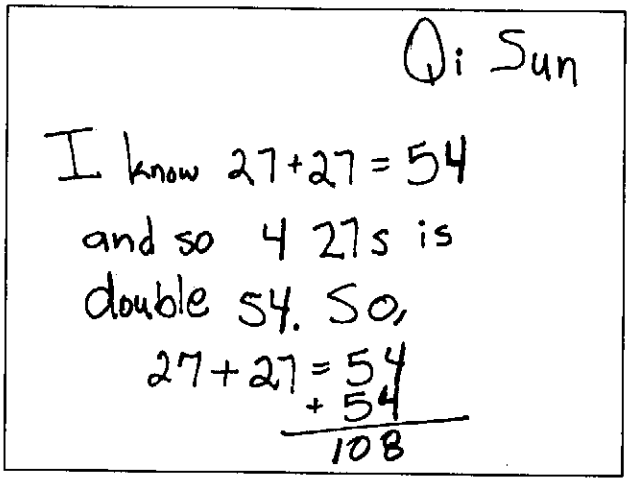
- *Addition Strategies.* Many students see this problem as one of repeated addition.



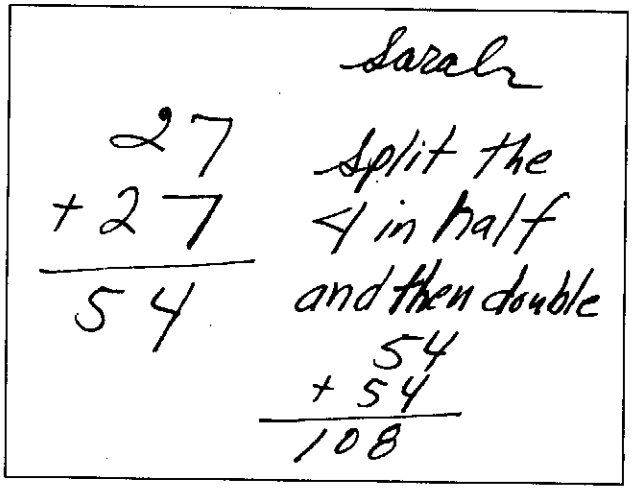
Some students used the same notation but explained that they "skip counted by 27's."



Still others may know that if two 27's is equal to 54 then four 27's is double that. One student represented the problem in this way:



Similarly, another student wrote,



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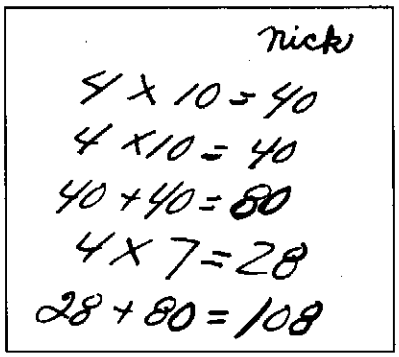
Teacher Note

continued

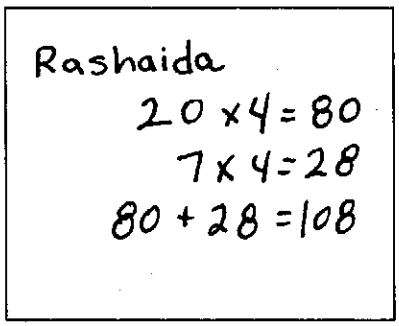
- *Multiplication Strategies.* We expect that many students will be able to break apart the problem into smaller, more familiar multiplication problems as one of their strategies. Here is how one student kept track of each part of the problem and of the totals of those problems:



Some students will break 27 into 20 + 7, multiply each by 4, and add the subtotals:

> "I counted 20 four times and got 80. Then I said 7 × 4 and got 28, and then I added them together and got 108."

Here is another version of a student's record of this solution:



DIALOGUE BOX

Strategies for Learning "Hard Problems"

This discussion occurred while students were working on the activity Which Pairs Are Hard for You? (p. 55).

DeShane: I think 6 × 8 is a hard problem. It's on my list every time.

[Lots of students groan and agree with DeShane. A few students in the class disagree.]

Teresa: I had 6 × 8 on my list too, but you can think of it like 6 × 4 and 6 × 4. That's 24 plus 24, and that's easier to figure out for me. It's like the Small Array/Big Array game. 6 × 4 is the small array and 6 × 8 is the big array, so it's a match.

Rashaida: I just think of 5 × 8 which I know is equal to 40, then I add 8 more.

DeShane: That's hard.

Qi Sun: I just memorized it. I kept thinking, 6 × 8 is equal to 48, 6 × 8 is equal to 48; then I just remembered it.

Those are some interesting strategies for remembering a hard problem. Teresa broke the problem down into pieces she knew. Rashaida connected it to a fact she already knew and then added on, and Qi Sun practiced it over and over again. Any other ideas? How about skip counting?,

Sarah: You could skip count by 8's, but that's tricky.

DeShane: Or you could do 6's, that's tricky too. I think I'll try skip counting by 6's with my partner today at Choice Time.

Sounds like a good idea.